Synthesis of a quantum circuit with unique two-qubit layers

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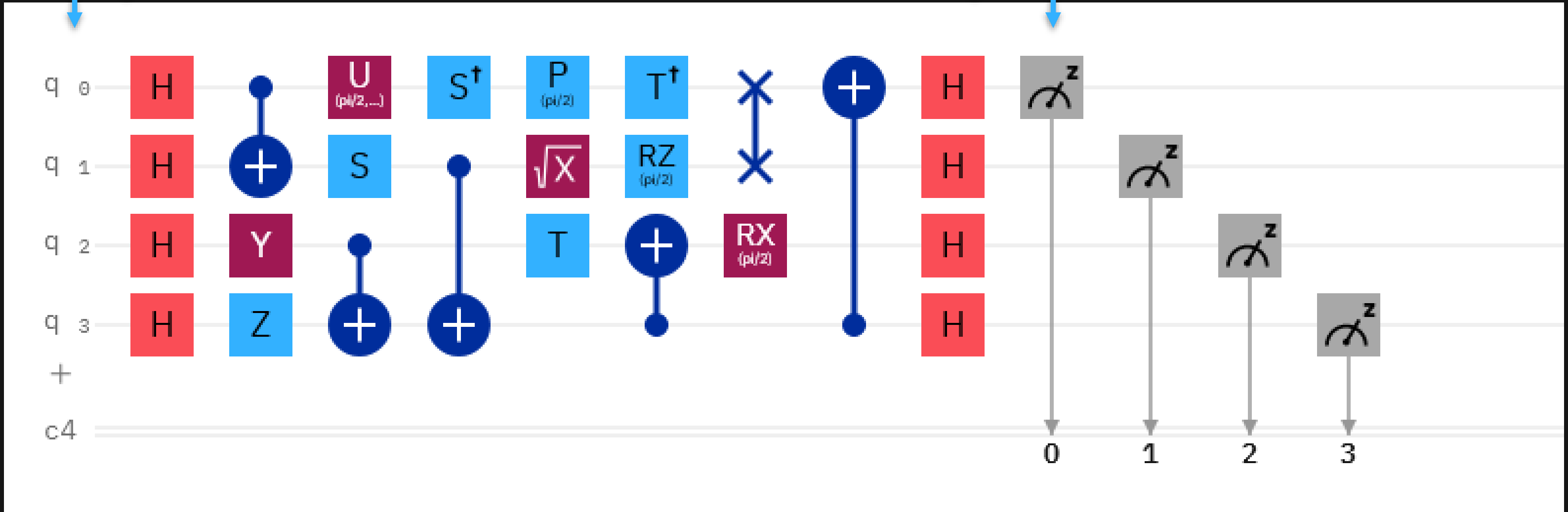
IBM **Quantum**

Quantum circuits

qubits

1-qubit and 2-qubit quantum gates

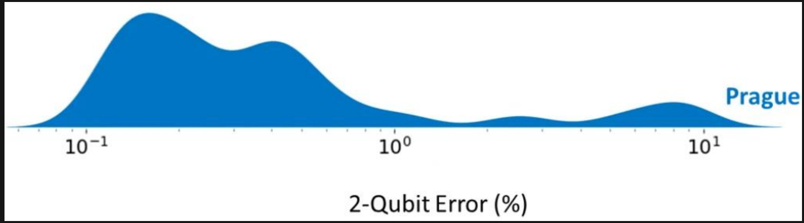
measurement



https://quantum-computing.ibm.com/

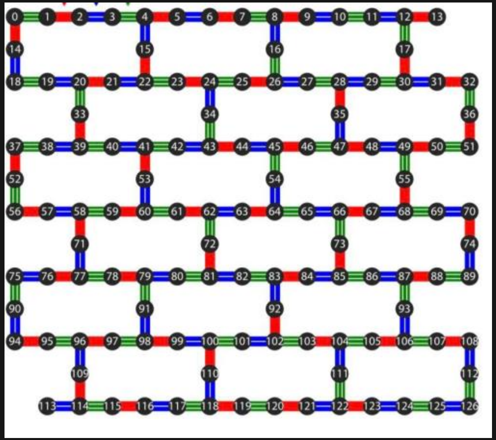
Quantum circuit synthesis challenges

- Quantum gates are noisy
  - 2-qubit gates are 10x noisier than 1-qubit gates
  - In Falcon R10 most 2-qubit gates approach 99.9% fidelity



https://research.ibm.com/blog/quantum-volume-256

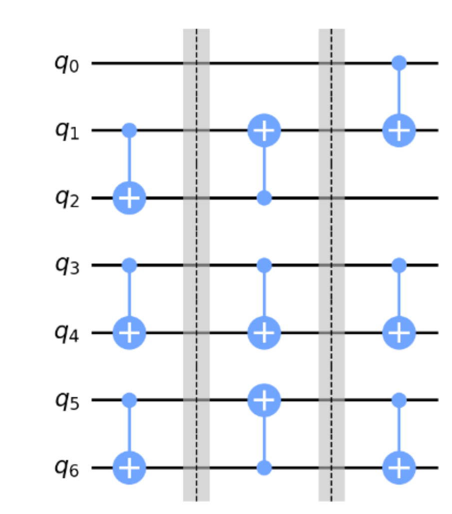
- The quantum device has restricted connectivity
  - 1 SWAP gate = 3 CNOT gates



https://research.ibm.com/blog/heavy-hex-lattice

Quantum circuit synthesis optimization

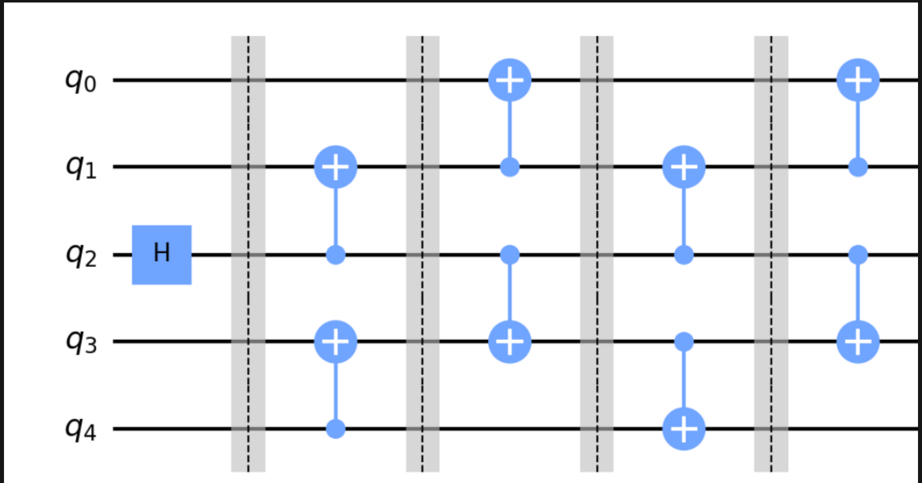
1. Minimize the number of 2-qubit gates
2. Minimize the 2-qubit depth, i.e. the number of 2-qubit gate layers
3. Minimize the number of unique 2-qubit gate layers



Example: quantum circuit with 3 CX layers and 2 unique layers

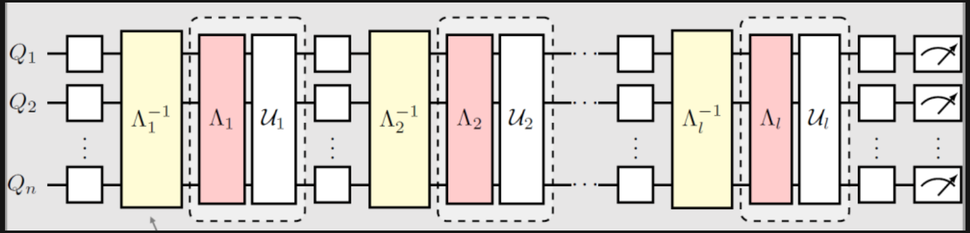
Example
Checkerboard state on a line connectivity

The checkerboard state $\frac{1}{\sqrt{2}}|00000\rangle + \frac{1}{\sqrt{2}}|10101\rangle$ with 4 CX layers and 2 unique CX layers

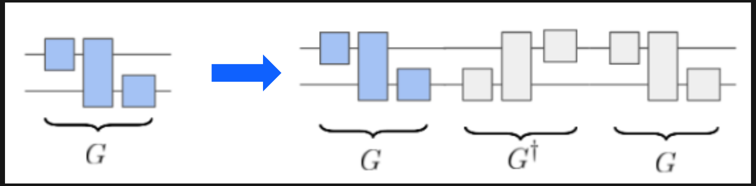


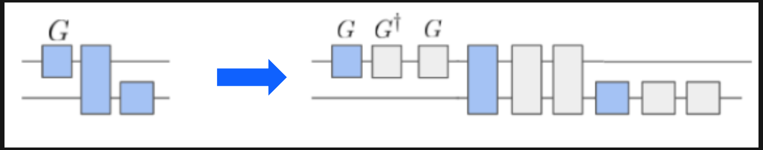
Motivation
Advanced mitigation methods

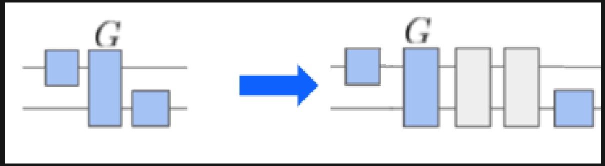
Probabilistic Error Cancellation (PEC) [1,2]



Zero Noise Extrapolation (ZNE) [1,3]







[1] Temme, Bravyi and Gambetta, *PRL*, 2017
[2] van den Berg, Minaev, Kandala, arxiv:2201.09866
[3] Kandala, Temme and Corcoles, *Nature*, 2019

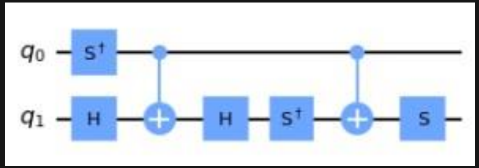
Motivation
Advanced mitigation methods

➢ PEC and ZNE require learning the noise-model for each unique gate layer

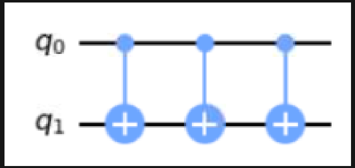
- Learning the Pauli noise models takes a lot of time for each layer
- The noise models can drift after some time
- Simplified control and calibration for these layers

Multiplying CX gates

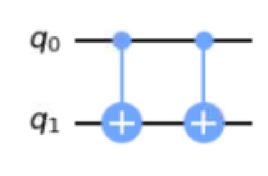
- A CX gate can be written using 2 CX gates [1,2]

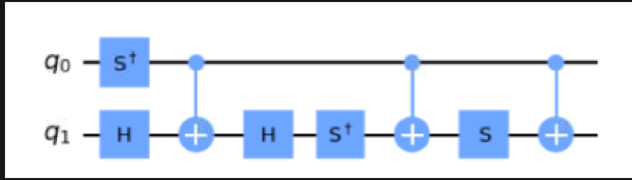


- A CX gate can also be written using 3 CX gates



- A 2-qubit identity gate can be written using 2 CX gates or 3 CX gates

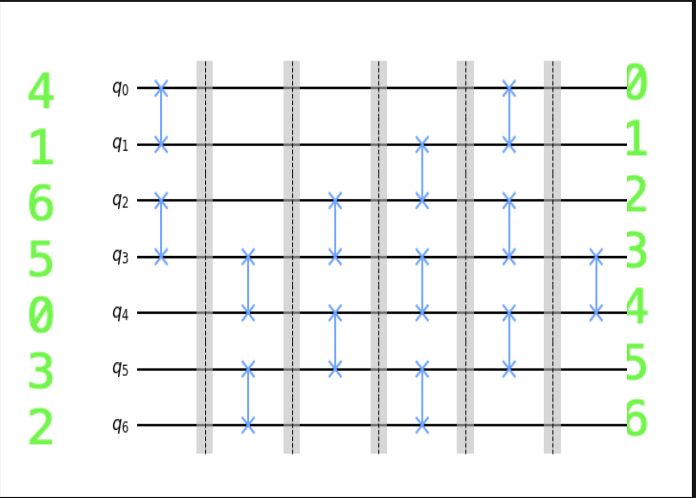




[1] Barenco et al. *Phys. Rev. A,* 1995
[2] Shende, Bullock, Markov, *IEEE Trans. CAD,* 2006

Permutation circuits

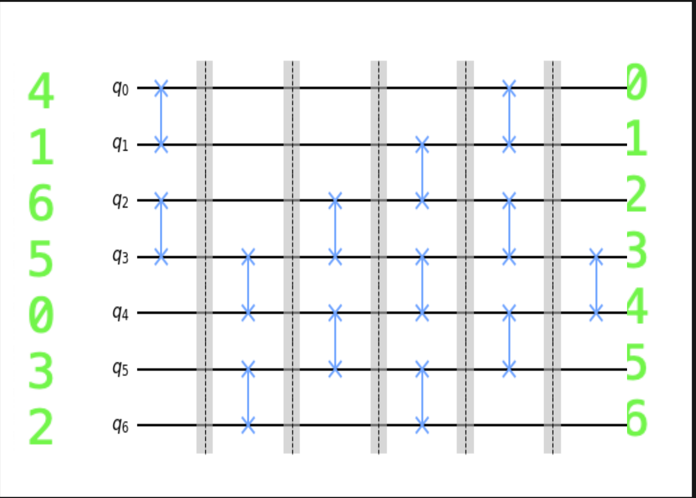
- A Permutation Circuit is a n qubit circuit containing only SWAP gates
- Any permutation circuit can be decomposed into SWAP gates in depth n for a line connectivity using a sorting network [1]



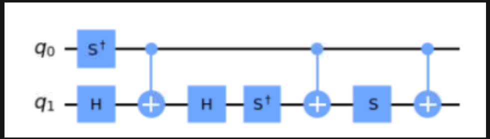
[1] Kutin, Moulton, Smithline, *Chicago Journal, 2007*
https://qiskit.org/documentation/stubs/qiskit.synthesis.synth_permutation_depth_lnn_kms.html

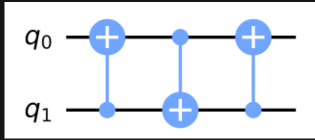
Permutation circuits

- A Permutation Circuit is a n qubit circuit containing only SWAP gates
- Any permutation circuit can be decomposed into CX gates in depth 3n for a line connectivity with 2 unique layers (odd/even layers)



SWAP → 3 CX gates
ID → 3 CX gates





[1] Kutin, Moulton, Smithline, *Chicago Journal, 2007*

Linear circuits

- A Linear Circuit is a n qubit circuit containing only CNOT gates (and resulting composite gates, such as SWAP)
- Any linear circuit can be decomposed in depth 5n for a line connectivity [1]

Sorting net of depth 2n odd/even layers

Sorting net of depth 3n odd/even layers

[1] Kutin, Moulton, Smithline, *Chicago Journal, 2007*
https://qiskit.org/documentation/stubs/qiskit.synthesis.synth_cnot_depth_line_kms.html

Linear circuits

- A Linear Circuit is a n qubit circuit containing only CNOT gates (and resulting composite gates, such as SWAP)
- Any linear circuit can be decomposed in depth 5n for a line connectivity with 2 unique layers

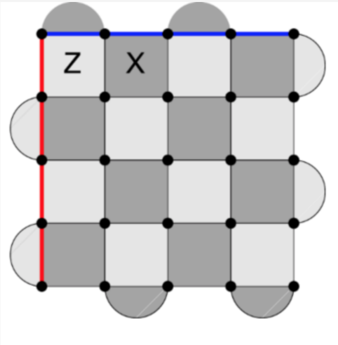
Sorting net of depth 2n odd/even layers of 0/1/2 CX gates

Sorting net of depth 3n odd/even layers of 2/3 CX gates

[1] Kutin, Moulton, Smithline, *Chicago Journal, 2007*

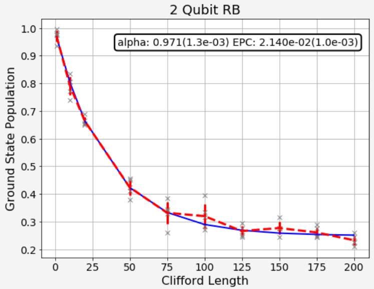
Clifford circuits

Quantum error correction codes



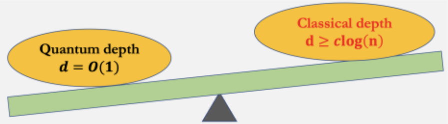
Clifford gates are used in stabilizer codes and surface codes

Randomized Benchmarking



Characterize quantum gate error

Quantum advantage with shallow circuits



Constant depth Clifford circuits can solve certain problems that constant depth classical circuits cannot

Bravyi, Gosset, Koenig *Science*, 2018

Clifford circuits

- The Clifford Circuits are generated by the quantum gates: *H*, *S* and *CX*
- Decomposing the Clifford circuit into layers [1]

H – S – CZ – CX – H – S – CZ – Pauli

- Optimized algorithms for the CZ and CZ – CX subcircuits for a line connectivity, the n-qubit Clifford 2-qubit depth is 7n-4
  - Depth of an n-qubit CZ – CX circuit is bounded by 5n [2]
  - Depth of an n-qubit CZ circuit is bounded by 2n+2 [3]
  - Local optimizations reduce 6 layers [2]

https://qiskit.org/documentation/stubs/qiskit.synthesis.synth_clifford_depth_lnn.html

[1] Bravyi and Maslov, *IEEE Trans. Info. Theory*, 2021
[2] Maslov and Yang, arxiv:2210:16195, 2022
[3] Maslov and Roetteler, *IEEE Trans. Info. Theory*, 2018

Clifford circuits

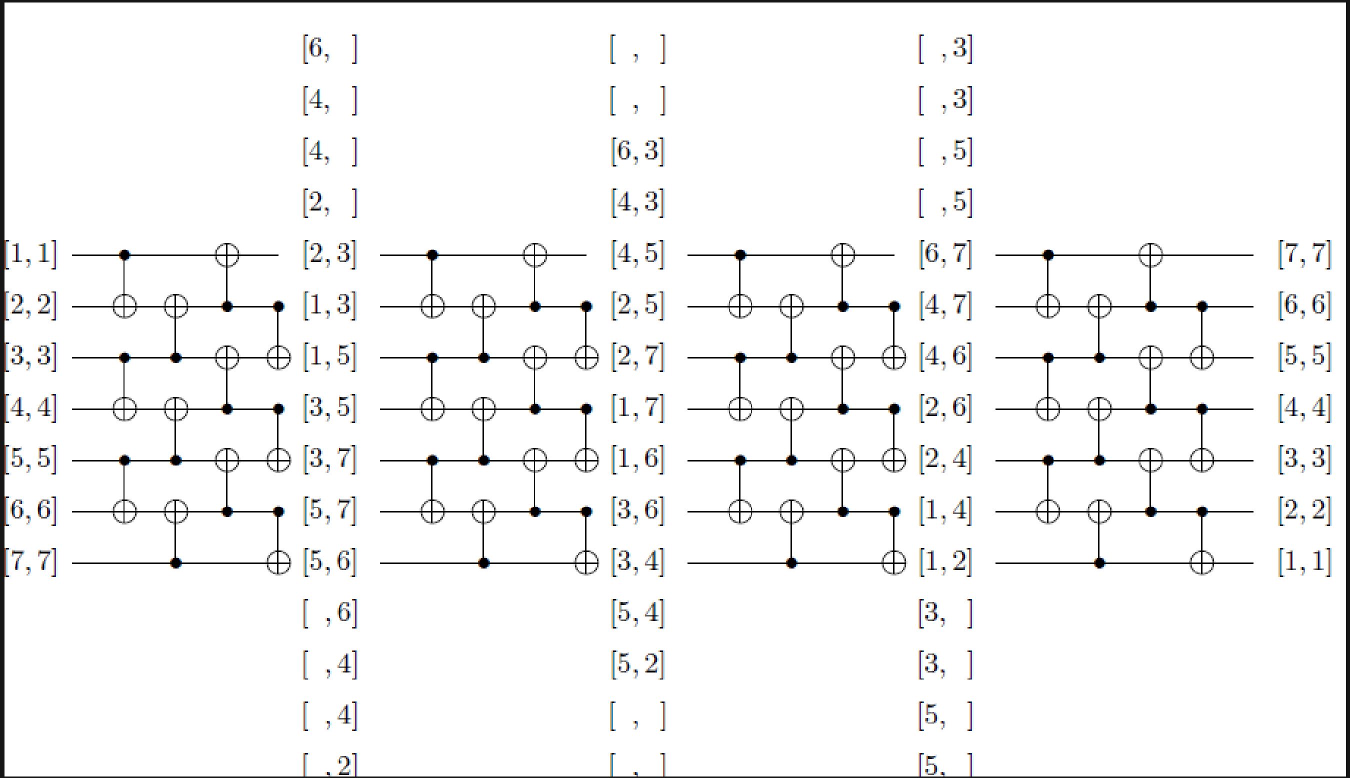
- The Clifford Circuits are generated by the quantum gates: *H*, *S* and *CX*
- Decomposing the Clifford circuit into layers [1]

H – S – CZ – CX – H – S – CZ – Pauli

- Optimized algorithms for the CZ and CZ – CX subcircuits for a line connectivity, the n-qubit Clifford 2-qubit depth is 7n-2 with 2 unique layers
  - Depth of an n-qubit CZ – CX circuit is bounded by 5n with 2 unique layers
  - Depth of an n-qubit CZ circuit is bounded by 2n+2 with 2 unique layers [3]
  - Local optimizations reduce 4 layers

[1] Bravyi and Maslov, *IEEE Trans. Info. Theory*, 2021
[2] Maslov and Yang, arxiv:2210:16195, 2022
[3] Maslov and Roetteler, *IEEE Trans. Info. Theory*, 2018

CZ circuits



Maslov and Roetteler, *IEEE Trans. Info. Theory*, 2018

https://qiskit.org/documentation/stubs/qiskit.synthesis.synth_cz_depth_line_mr.html

Stabilizer circuits

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- Decomposing the Clifford circuit into layers [1]

H – S – CZ – CX	–	H – S – CZ – Pauli
Preserves the ground state		Stabilizer circuit

- A CZ circuit can be decomposed in depth 2n+2 for a line connectivity with 2 unique layers [2]

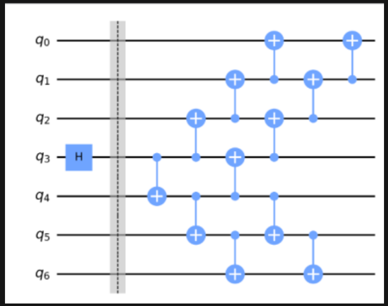
- Hence, a stabilizer circuit can be decomposed in depth 2n+2 for a line connectivity with 2 unique layers

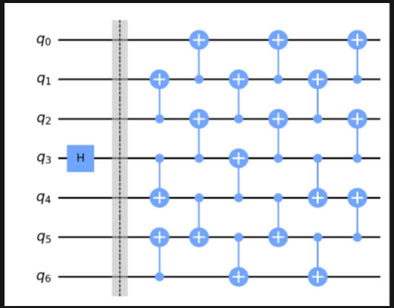
- [1] Bravyi and Maslov, *IEEE Trans. Info. Theory*, 2021
  [2] Maslov and Roetteler, *IEEE Trans. Info. Theory*, 2018

https://qiskit.org/documentation/stubs/qiskit.synthesis.synth_stabilizer_depth_lnn.html

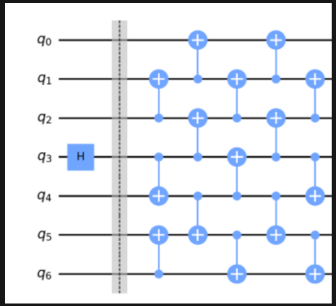
GHZ and checkerboard states on a line connectivity

The n-qubit GHZ state $\frac{1}{\sqrt{2}}|0 \ldots 0\rangle + \frac{1}{\sqrt{2}}|1 \ldots 1\rangle$ and checkerboard state $\frac{1}{\sqrt{2}}|0 \ldots 0\rangle + \frac{1}{\sqrt{2}}|10 \ldots 101\rangle$ can be written with ~n/2 CX layers and 2 unique CX layers on a line connectivity





Checkerboard state



GHZ state

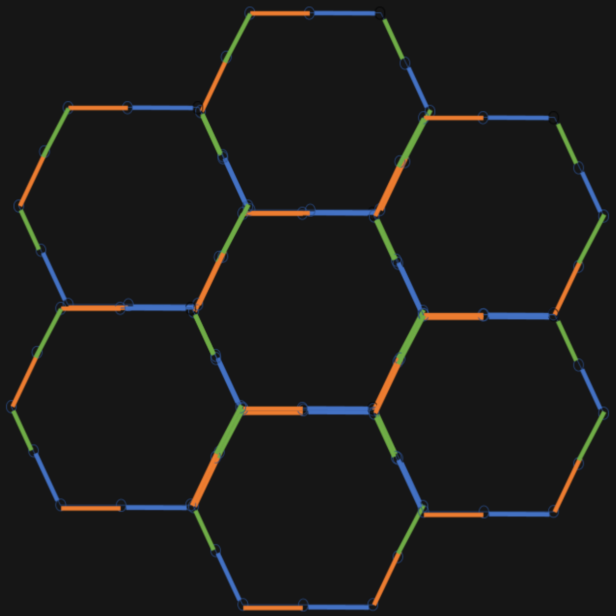
From a line connectivity to a general graph

- What is the minimal number of distinct layers?
- Each unique CX layer corresponds to a color of the corresponding edges
- Vizing Theorem. Every simple undirected graph with degree at most d can be edge colored by at most d+1 colors
- In some cases, d colors are enough
  - Line → 2 colors
  - Hex / Heavy-hex → 3 colors
  - Grid / Heavy-grid → 4 colors

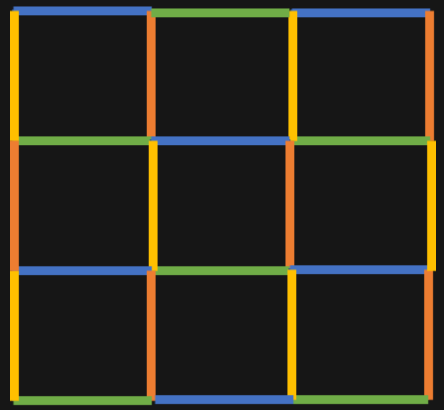
Edge coloring

Line
2 colors

Heavy-hex
3 colors



Grid
4 colors



Summary

- Motivated by advanced mitigation methods (like PEC and ZNE) we aim to reduce the number of unique CX layers
- Many circuits can be synthesized with 2 unique CX layers on a line connectivity w/o increasing the circuit total depth
  - Permutations and linear circuits
  - Clifford circuits and stabilizer states
  - GHZ and checkerboard states
- The latter can also be generalized to arbitrary connectivity maps

Thank you !

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