Synthesis of a quantum circuit with unique two-qubit layers

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Quantum circuits

1-qubit and 2-qubit quantum gates

measurement

qubits

https://quantum-computing.ibm.com/
Quantum circuit synthesis challenges

➢ Quantum gates are noisy
  • 2-qubit gates are 10x noisier than 1-qubit gates
  • In Falcon R10 most 2-qubit gates approach 99.9% fidelity

➢ The quantum device has restricted connectivity
  • 1 SWAP gate = 3 CNOT gates

https://research.ibm.com/blog/quantum-volume-256

https://research.ibm.com/blog/heavy-hex-lattice
Quantum circuit synthesis optimization

1. Minimize the number of 2-qubit gates

2. Minimize the 2-qubit depth, i.e. the number of 2-qubit gate layers

3. Minimize the number of unique 2-qubit gate layers

Example: quantum circuit with 3 CX layers and 2 unique layers
Example
Checkerboard state on a line connectivity

The checkerboard state $\frac{1}{\sqrt{2}} |00000\rangle + \frac{1}{\sqrt{2}} |10101\rangle$
with 4 CX layers and 2 unique CX layers
Motivation
Advanced mitigation methods

Probabilistic Error Cancellation (PEC) [1,2]

Zero Noise Extrapolation (ZNE) [1,3]

Motivation

Advanced mitigation methods

➢ PEC and ZNE require learning the noise-model for each unique gate layer
  • Learning the Pauli noise models takes a lot of time for each layer
  • The noise models can drift after some time
  • Simplified control and calibration for these layers
Multiplying CX gates

➢ A CX gate can be written using 2 CX gates \[^{[1,2]}\]

➢ A CX gate can also be written using 3 CX gates

➢ A 2-qubit identity gate can be written using 2 CX gates or 3 CX gates

A **Permutation Circuit** is a \( n \) qubit circuit containing only SWAP gates.

Any permutation circuit can be decomposed into SWAP gates in depth \( n \) for a line connectivity using a sorting network \([1]\).
Permutation circuits

- A Permutation Circuit is a n qubit circuit containing only SWAP gates.
- Any permutation circuit can be decomposed into CX gates in depth $3n$ for a line connectivity with 2 unique layers (odd/even layers).

A Linear Circuit is a n qubit circuit containing only CNOT gates (and resulting composite gates, such as SWAP)

Any linear circuit can be decomposed in depth $5n$ for a line connectivity [1]

https://qiskit.org/documentation/stubs/qiskit.synthesis.synth_cnot_depth_line_kms.html
A Linear Circuit is a n qubit circuit containing only CNOT gates (and resulting composite gates, such as SWAP)

Any linear circuit can be decomposed in depth $5n$ for a line connectivity with 2 unique layers

Clifford circuits

Quantum error correction codes

Clifford gates are used in stabilizer codes and surface codes

Randomized Benchmarking

Characterize quantum gate error

Quantum advantage with shallow circuits

Constant depth Clifford circuits can solve certain problems that constant depth classical circuits cannot

Bravyi, Gosset, Koenig

Science, 2018
Clifford circuits

➢ The Clifford Circuits are generated by the quantum gates: $H$, $S$ and $CX$

➢ Decomposing the Clifford circuit into layers $^{[1]}$

$$H - S - CZ - CX - H - S - CZ - \text{Pauli}$$

➢ Optimized algorithms for the $CZ$ and $CZ - CX$ subcircuits for a line connectivity, the n-qubit Clifford 2-qubit depth is $7n-4$

   • Depth of an n-qubit $CZ - CX$ circuit is bounded by $5n$ $^{[2]}$

   • Depth of an n-qubit $CZ$ circuit is bounded by $2n+2$ $^{[3]}$

   • Local optimizations reduce 6 layers $^{[2]}$


$^{[2]}$ Maslov and Yang, arxiv:2210:16195, 2022

Clifford circuits

- The **Clifford Circuits** are generated by the quantum gates: $H$, $S$ and $CX$
- Decomposing the Clifford circuit into layers $^[1]$
  
  
  $$H - S - CZ - CX - H - S - CZ - \text{Pauli}$$

- Optimized algorithms for the $CZ$ and $CZ - CX$ subcircuits for a line connectivity, the n-qubit Clifford 2-qubit depth is $7n-2$ with 2 unique layers
  
  - Depth of an n-qubit $CZ - CX$ circuit is bounded by $5n$ with 2 unique layers
  
  - Depth of an n-qubit $CZ$ circuit is bounded by $2n+2$ with 2 unique layers $^[3]$
  
  - Local optimizations reduce 4 layers


CZ circuits


[Link to Qiskit documentation](https://qiskit.org/documentation/stubs/qiskit.synthesis.synth_cz_depth_line_mr.html)
Stabilizer circuits

- Decomposing the Clifford circuit into layers [1]
  
  $H - S - CZ - CX$ - $H - S - CZ - Pauli$

  Preserves the ground state

  Stabilizer circuit

- A CZ circuit can be decomposed in depth $2n+2$ for a line connectivity with 2 unique layers [2]

- Hence, a stabilizer circuit can be decomposed in depth $2n+2$ for a line connectivity with 2 unique layers


https://qiskit.org/documentation/stubs/qiskit.synthesis.synth_stabilizer_depth_lnn.html
GHZ and checkerboard states on a line connectivity

The n-qubit GHZ state $\frac{1}{\sqrt{2}} |0 \ldots 0\rangle + \frac{1}{\sqrt{2}} |1 \ldots 1\rangle$ and checkerboard state $\frac{1}{\sqrt{2}} |0 \ldots 0\rangle + \frac{1}{\sqrt{2}} |10 \ldots 101\rangle$ can be written with $\sim n/2$ CX layers and 2 unique CX layers on a line connectivity.
From a line connectivity to a general graph

➢ What is the minimal number of distinct layers?
➢ Each unique CX layer corresponds to a color of the corresponding edges
➢ **Vizing Theorem.** Every simple undirected graph with degree at most $d$ can be edge colored by at most $d+1$ colors
➢ In some cases, $d$ colors are enough
  • Line $\rightarrow$ 2 colors
  • Hex / Heavy-hex $\rightarrow$ 3 colors
  • Grid / Heavy-grid $\rightarrow$ 4 colors
Edge coloring

Line
2 colors

Heavy-hex
3 colors

Grid
4 colors
Motivated by advanced mitigation methods (like PEC and ZNE) we aim to reduce the number of unique CX layers.

Many circuits can be synthesized with 2 unique CX layers on a line connectivity w/o increasing the circuit total depth:

- Permutations and linear circuits
- Clifford circuits and stabilizer states
- GHZ and checkerboard states

The latter can also be generalized to arbitrary connectivity maps.
Thank you!

https://quantum-computing.ibm.com