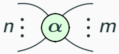
# Speedy Contraction of ZX Diagrams with Triangles

**Mark Koch** Quantinuum
Richie Yeung University of Oxford, Quantinuum
Quanlong Wang Quantinuum

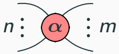
July 22, 2023

IWQC 2023

# ZX Contraction




$$= |0\rangle^m \langle 0|^n + e^{i\alpha} |1\rangle^m \langle 1|^n$$




$$= |+\rangle^m \langle +|^n + e^{i\alpha} |-\rangle^m \langle -|^n$$

$$= |+\rangle\langle 0| + |-\rangle\langle 1|$$

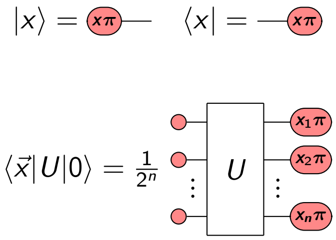
Diagram with $n$ inputs, $m$ outputs $\rightsquigarrow$ $\mathbb{C}^{2^m \times 2^n}$

Diagram without inputs/outputs $\rightsquigarrow$ $\mathbb{C}$

**Contraction:** Given scalar ZX diagram, compute its value

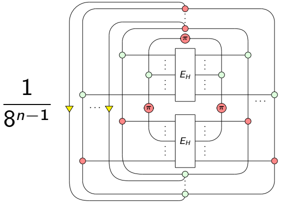
# Applications

## Classical Simulation




## Quantum ML

(ZXW Calculus)

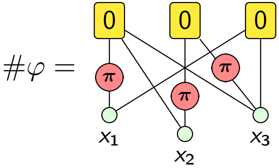



$$= \mathrm{Var}\left(\frac{\partial\langle H\rangle}{\partial\theta}\right)$$

## Counting SAT

(ZH Calculus)

$$\varphi = (x_1 \vee x_3) \wedge (\neg x_2 \vee \neg x_3) \wedge (\neg x_1 \vee x_2 \vee \neg x_3)$$



## How to Contract

Tensor network based methods

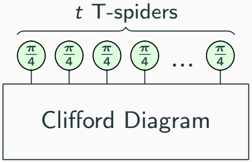
Stabiliser decompositions**Stabiliser decompositions**

- Introduced in [Bravyi and Gosset, 2016]
- Adapted to ZX in [Kissinger and van de Wetering, 2022] and [Kissinger et al., 2022]
- Implemented in QuiZX
- Roughly: Clifford diagrams are easy to contract (Gottesman-Knill)

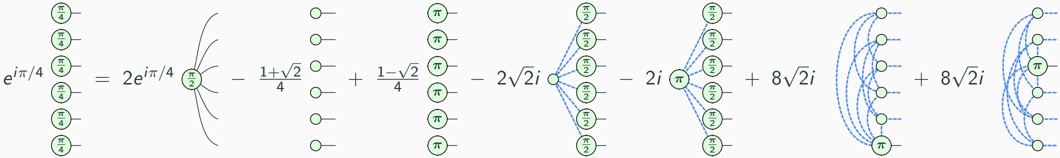
  $\Rightarrow$ Decompose non-Clifford diagram into sum of Clifford diagrams

# ZX Stabiliser Decompositions [Kissinger and van de Wetering, 2022]

1. Given a **Clifford+T** diagram




2. Pick 6 T-spiders and decompose them via [Bravyi et al., 2019]




We get 7 diagrams with $t-6$ Ts each

# ZX Stabiliser Decompositions [Kissinger and van de Wetering, 2022]

3. Simplify each diagram to reduce T-count
4. Recurse: For each diagram, pick 6 new T-spiders and decompose

$$\Rightarrow \text{now } 7^2 \text{ diagrams} \quad \Rightarrow 7^3 \text{ diagrams} \quad \Rightarrow \ldots$$

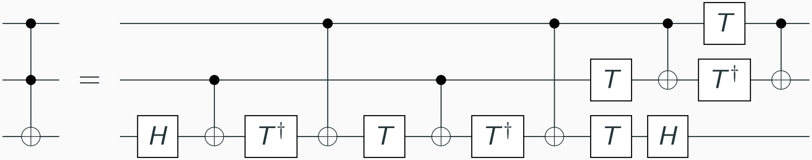
5. Once no more T-spiders left: Contract remaining Clifford diagrams via Gottesman-Knill and add up results

$\Rightarrow 7^{t/6}$ terms $= 2^{\alpha t}$ terms where $\alpha = \frac{\log(7)}{6} \approx 0.468$

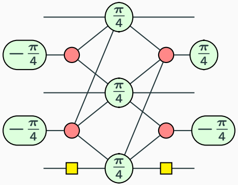
Best known decomp has $\alpha \approx 0.396$ [Kissinger et al., 2022]

# Our Approach

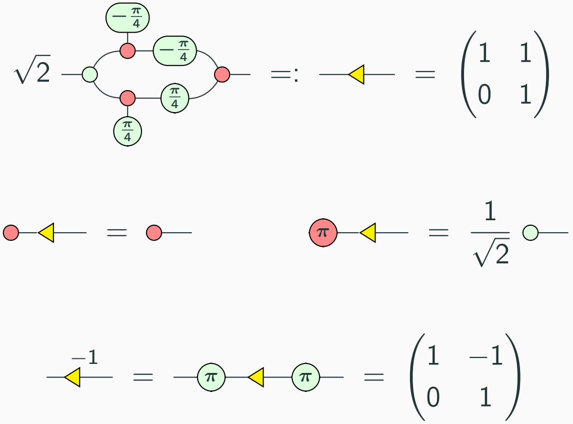
# Motivation



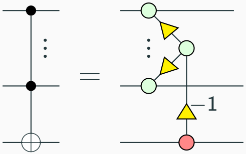

$= 4\sqrt{2}$

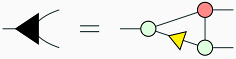


# Triangle



# Triangle



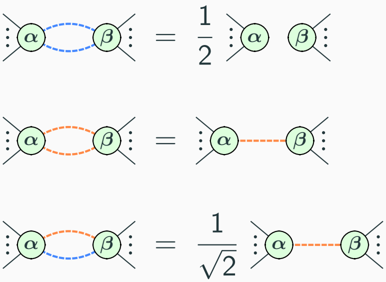



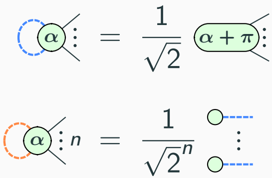
⇒ Apply stabiliser decompositions to triangles

# Graph-Like

Only green spiders and Hadamard edges **or triangle edges**

Remove parallel edges / self-loops





# Decomposing Triangle Edges

# Naive Baseline

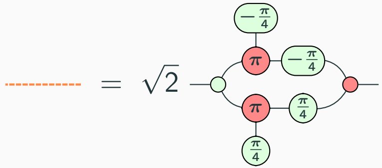


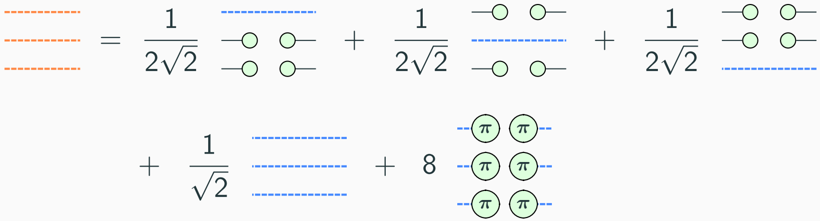

Diagram with $t$ triangles $\Rightarrow$ Diagram with $4t$ T-spiders

$2^{\beta t}$ terms, $\beta = 4\alpha \approx 1.584$

# Single Triangle

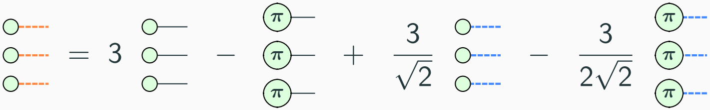

Scales with $\beta = 1$

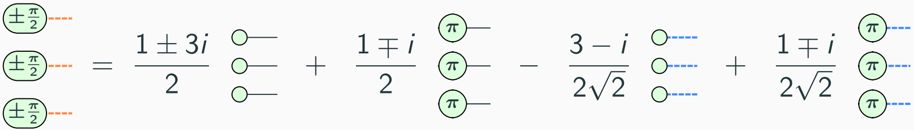
# Multiple Triangles at Once




Scales with $\beta = \frac{\log(5)}{3} \approx 0.774$

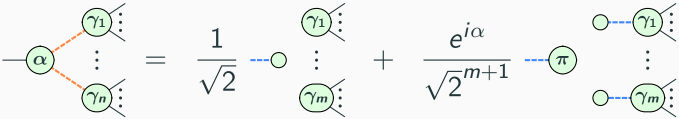
# Special Cases



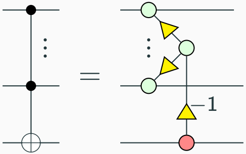




Scales with $\beta = \frac{2}{3} \approx 0.667$

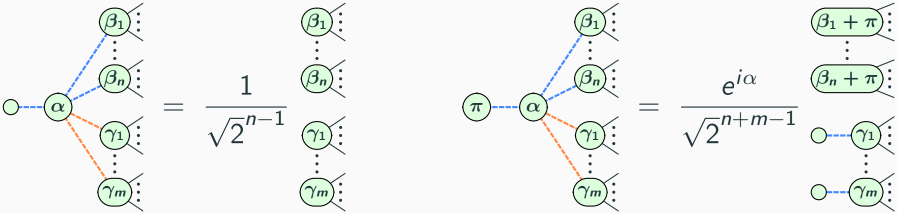
# Best Case



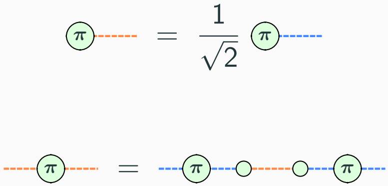

Scales with $\beta = \frac{1}{n}$



# Simplification

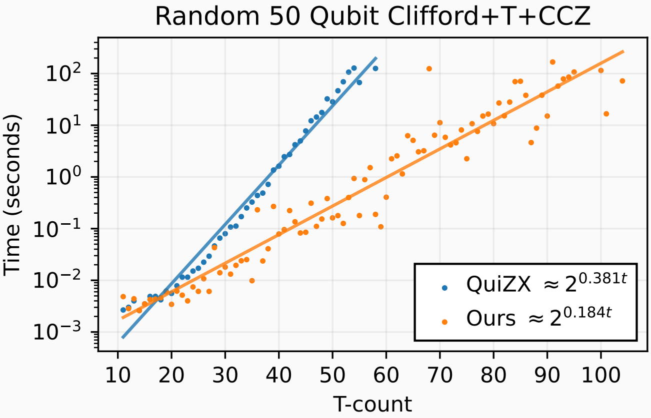


# Simplification

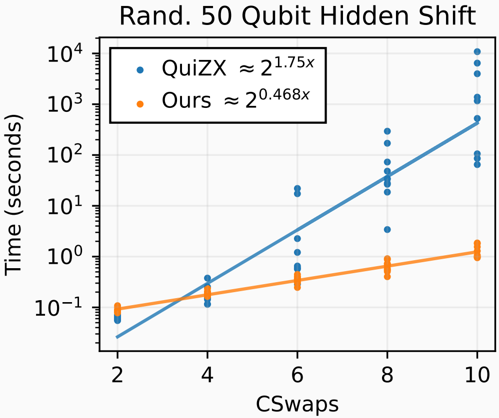


# Benchmarks

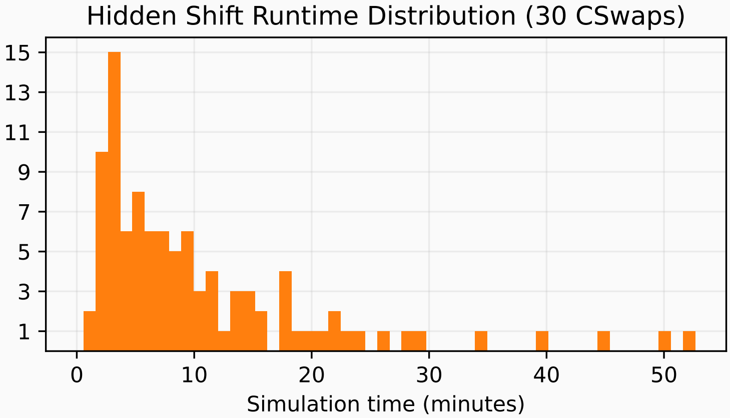
# Random Circuits



# Hidden Shift



# Hidden Shift



# Barren Plateau Detection

# Barren Plateaus

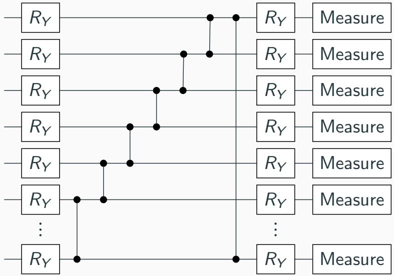
## Problem

Gradient landscape of many parametrised circuits is exponentially flat

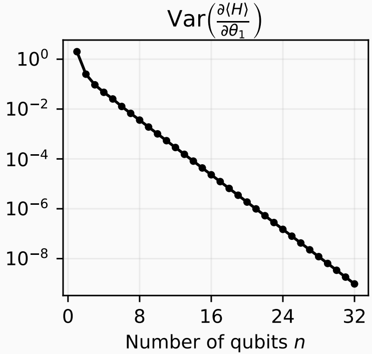
Typically: $E\left(\frac{\partial\langle H\rangle}{\partial\theta_j}\right) = 0$

$\Rightarrow \mathrm{Var}\left(\frac{\partial\langle H\rangle}{\partial\theta_j}\right) \approx 0$ is bad news

# Example

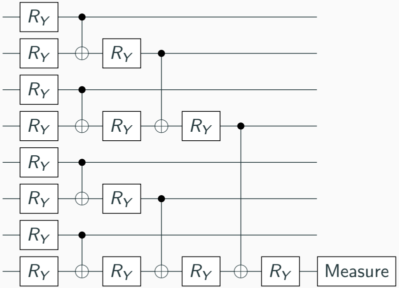


$R_Y$
Measure

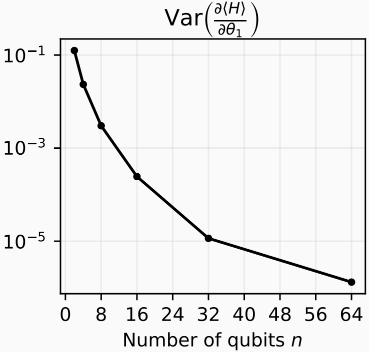



$\mathrm{Var}\left(\frac{\partial\langle H\rangle}{\partial\theta_1}\right)$
$10^0$
$10^{-2}$
$10^{-4}$
$10^{-6}$
$10^{-8}$
0
8
16
24
32
Number of qubits $n$

# Example



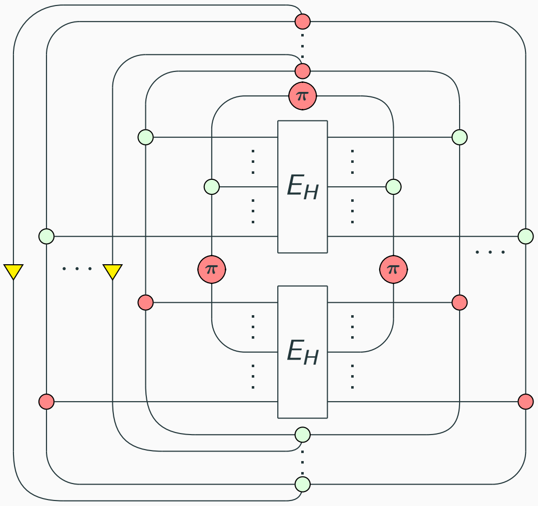
$R_Y$
Measure

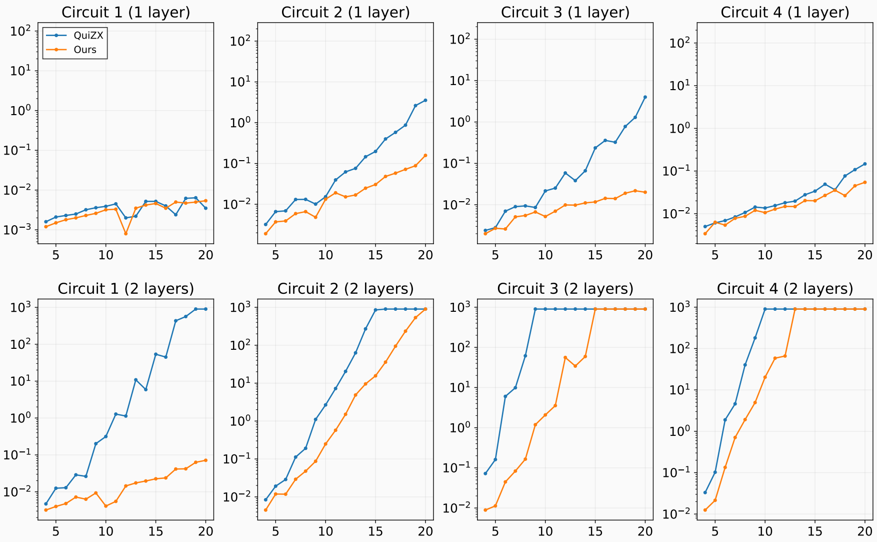
$\mathrm{Var}\left(\frac{\partial \langle H \rangle}{\partial \theta_1}\right)$
$10^{-1}$
$10^{-3}$
$10^{-5}$
0
8
16
24
32
40
48
56
64
Number of qubits $n$

# Gradient Variance in ZX [Wang et al., 2022]

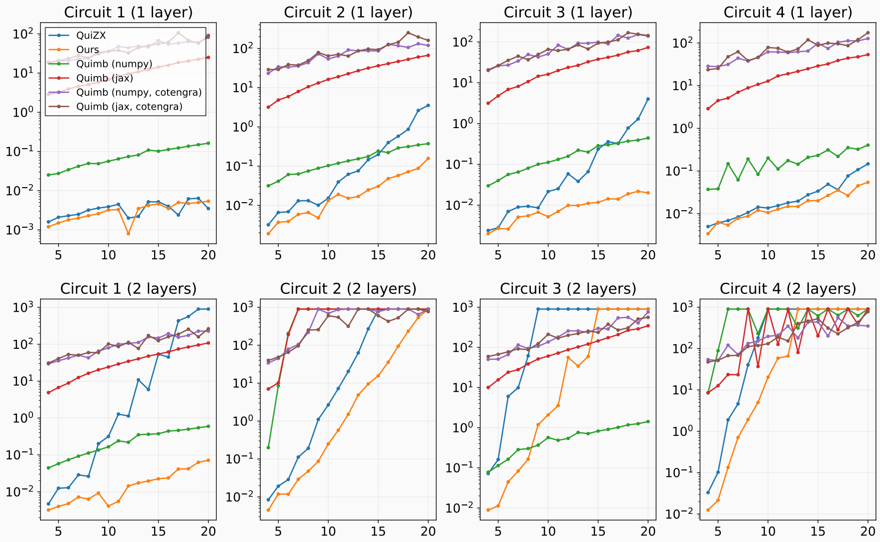
$$\mathrm{Var}\left(\frac{\partial \langle H \rangle}{\partial \theta_j}\right) = \frac{1}{8^{n-1}}$$



# Benchmark


Circuit 1 (1 layer)
Circuit 2 (1 layer)
Circuit 3 (1 layer)
Circuit 4 (1 layer)
Circuit 1 (2 layers)
Circuit 2 (2 layers)
Circuit 3 (2 layers)
Circuit 4 (2 layers)
QuiZX
Ours

# Benchmark



Circuit 1 (1 layer)
Circuit 2 (1 layer)
Circuit 3 (1 layer)
Circuit 4 (1 layer)
Circuit 1 (2 layers)
Circuit 2 (2 layers)
Circuit 3 (2 layers)
Circuit 4 (2 layers)
QuiZX
Ours
Quimb (numpy)
Quimb (jax)
Quimb (numpy, cotengra)
Quimb (jax, cotengra)

# Summary

We contract ZX diagrams with triangles via stabiliser decompositions to

1. Speed up simulation of multi-controlled gates
2. Contract ZXW diagrams (e.g. barren plateau detection)

Future Work:

- Hamiltonians in ZXW
- Better triangle decompositions?
- Approximate contraction?

# Thank you!

arXiv:2307.01803

# References i


Bravyi, S., Browne, D., Calpin, P., Campbell, E., Gosset, D., and Howard, M. (2019).
**Simulation of quantum circuits by low-rank stabilizer decompositions.**
*Quantum*, 3:181.

Bravyi, S. and Gosset, D. (2016).
**Improved classical simulation of quantum circuits dominated by clifford gates.**
*Phys. Rev. Lett.*, 116:250501.

Kissinger, A. and van de Wetering, J. (2022).
**Simulating quantum circuits with zx-calculus reduced stabiliser decompositions.**
*Quantum Science and Technology*, 7(4):044001.

Kissinger, A., van de Wetering, J., and Vilmart, R. (2022).
**Classical Simulation of Quantum Circuits with Partial and Graphical Stabiliser Decompositions.**
In Le Gall, F. and Morimae, T., editors, *17th Conference on the Theory of Quantum Computation, Communication and Cryptography (TQC 2022)*, volume 232 of *Leibniz International Proceedings in Informatics (LIPIcs)*, pages 5:1–5:13, Dagstuhl, Germany. Schloss Dagstuhl – Leibniz-Zentrum für Informatik.

# References ii


Wang, Q., Yeung, R., and Koch, M. (2022).
**Differentiating and integrating zx diagrams with applications to quantum machine learning.**
*arXiv preprint arXiv:2201.13250*.